Frustrated SU(4) as the Preonic Precursor
of the Standard Model

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ABSTRACT

We give a model for composite quarks and leptons based on the semisimple gauge group $SU(4)$, with the preons in the 10 representation; this choice of gauge gluon and preon multiplets is motivated by the possibility of embedding them in an $N = 6$ supergravity multiplet. Hypercolor singlets are forbidden in the fermionic sector of this theory; we propose that $SU(4)$ symmetry spontaneously breaks to $SU(3) \times U(1)$, with the binding of triality nonzero preons and gluons into composites, and with the formation of a color singlet condensate that breaks the initial Z_{12} vacuum symmetry to Z_6. The spin $1/2$ fermionic composites have the triality structure of a quark lepton family, and the initial Z_{12} symmetry implies that there are six massless families, which mix to give three distinct families, two massless with massive partners and one with both states massive, at the scale of the condensate. The spin 1 triality zero composites of the color triplet $SU(4)$ gluons, when coupled to the condensate and with the color singlet representation of the 10 acting as a doorway state, lead to weak interactions of the fermionic composites through an exact $SU(2)$ gauge algebra. The initial Z_{12} symmetry implies that this $SU(2)$ gauge algebra structure is doubled, which in turn requires that the corresponding independent gauge bosons must couple to chiral components of the composite fermions. Since the $U(1)$ couples to the 10 representation as $B - L$, an effective $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak theory arises at the condensate scale, with all composites having the correct electric charge structure. A renormalization group analysis shows that the conversion by binding of one 10 of $SU(4)$ to 12 triplets of $SU(3)$ can give a very large, calculable hierarchy ratio between the $SU(4)$ and the hadronic mass scales.

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1. Introduction

Although the repetition of quark-lepton families is strongly suggestive of composite structure, no plausible composite model has yet emerged, and the current focus of research on unification is based on the alternate idea of grand unification. In this paper we reexamine the idea of compositeness in the context of a new model for the formation of composites. Nearly all work to date on composites has assumed a "QCD-like" paradigm, in which the preons couple to a hypercolor force field, that acts independently of the standard model gauge fields (by a group theoretic direct product) and binds the preons into hypercolor singlets. Although based on well-studied physics, this approach suffers from a serious problem relating to chiral symmetry. In general, the direct product structure leads to a large global chiral symmetry group, even after the breaking of the overall $U(1)$ chiral symmetry by instantons. As a consequence, the 't Hooft anomaly matching conditions [1] must be obeyed if massless composites are to be possible, and extensive searches for solutions to these equations [2] show none that match the observed particle spectrum. When the 't Hooft conditions are not obeyed, a chiral symmetry breaking condensate must form on the binding scale of the theory, as happens in QCD; the composites then get large masses, and composite structure with a large hierarchy ratio is not possible.

We propose in this paper an alternative approach to composite structure, based on a partial grand unification using a hypercolor group and preonic fermion multiplet structure for which the hypercolor forces are *frustrated* in the fermionic sector, in the sense that hypercolor singlet fermionic states are forbidden. Together with a proposed chiral symmetry breaking chain, this leads to quark lepton composites (and further matter bound states of these), in which the impossibility of fermionic hypercolor singlets translates into the participation of

all fermionic composite states in spin-1 gluon mediated gauge interactions, as experimentally observed.

Although we do not make explicit use of supersymmetry in this paper, supersymmetry ideas are a principal motivation in the formulation of our model. Specifically, the hyper-color gauge group $SU(4)$, with the preons in a single 10 representation of Dirac fermions, are chosen for study because they correspond to the only $SU(N)$ based generalized "rishon" model that is embedable in an extended supergravity multiplet. Similarly, the chiral symmetry breaking chain that we postulate is strongly motivated by recent results of Seiberg [3] (for reviews see [4]) showing that there are supersymmetric systems in which preons with vector-like gauge couplings can form massless fermion composites, thus contradicting the "most attractive channel" rule for chiral symmetry breaking, and also showing that in supersymmetric systems massless composite non-Abelian gauge gluons can occur. With these recent results in mind, we propose a symmetry breaking route by which the known structure of the standard model emerges emerges from our preon model, with the correct family multiplicity, but (as distinguished from conventional grand unification) with the intermediate boson states arising as composites of three fundamental gluons. It is this latter feature that allows us to evade the usual grand unification restriction requiring a unification group of at least rank 4 (since the standard model is rank 4), and permits a hybrid grand-composite unification in the rank 3 group $SU(4)$. This in turn makes possible renewed consideration of the appealing idea that all matter and force carriers, including gravitation, may lie in a single extended supergravity multiplet.

2. Counting relations for generalized "rishon" models

Since our construction uses some of the basic notions of the Harari-Seiberg [5] "ris-

hon" model version of the Harari-Shupe [6] scheme, we briefly explain the relevant aspects here. The original Harari-Shupe scheme hoped to generate $SU(3)$ symmetry as a permutation symmetry acting on the preons, which is not possible within standard complex quantum field theory; moreover, despite an extensive investigation [7] that we have carried out of non-commutative quaternionic quantum mechanics, we still have found no concrete way to realize a dynamically generated exact color symmetry. So instead, following Harari and Seiberg, we will assume that the $SU(3)$ color group is present as a subgroup in the fundamental gauge interactions, which are treated in standard quantum field theory.

In the Harari-Seiberg model, the fundamental preons are postulated to be "rishon" states T and V, and their antiparticle states $\overline{T}$ and $\overline{V}$, with electric charges Q and $SU(3)$ trialities Tri assigned as follows:

$$Q(T) = 1/3, \quad Tri(T) = 1, \quad Q(\overline{T}) = -1/3, \quad Tri(\overline{T}) = -1$$
$$Q(V) = 0, \quad Tri(V) = -1, \quad Q(\overline{V}) = 0, \quad Tri(\overline{V}) = 1. \tag{1}$$

The quark and lepton states in the first family are then constructed as three preon composites according to the scheme

$$e^+ = TTT$$
$$u = TTV$$
$$\overline{d} = TVV \tag{2}$$
$$\nu = VVV \; ,$$

with the corresponding expressions for $e^-, \overline{u}, d, \overline{\nu}$ obtained by replacing T, V by $\overline{T}, \overline{V}$. Defining the particle number n_T, n_V, n_e, n_u, n_d, n_ν as the difference between the number of particles and antiparticles of the indicated type (counting e^-, as usual, as a particle) we

immediately find from Eqs. (1) and (2) the following counting relations

$$n_T = - 3n_e + 2n_u - n_d$$

$$n_V = 3n_\nu - 2n_d + n_u \quad ,$$

(3a)

from which we find

$$\frac{1}{3}(n_T - n_V) = \frac{1}{3}(n_u + n_d) - (n_e + n_\nu)$$

$$\frac{1}{3}(n_T + n_V) = n_u - n_d + n_\nu - n_e \; .$$

(3b)

Since Eq. (1) implies that the electric charge Q is given by

$$Q = \frac{1}{3}n_T = \frac{1}{6}(n_T + n_V) + \frac{1}{6}(n_T - n_V) \; ,$$

(4a)

if we define the baryon number B, lepton number L, and preon or fermion number F by

$$B = \frac{1}{3}(n_u + n_d)$$

$$L = n_e + n_\nu$$

$$F = n_T + n_V \; ,$$

(4b)

the electric charge can be rewritten as

$$Q = \frac{1}{6}F + \frac{1}{6}(n_T - n_V) = \frac{1}{6}F + \frac{1}{2}(B - L) \; .$$

(4c)

Finally, making the definitions

$$I_{3L} = \frac{1}{2}[n_{uL} - n_{dL} + n_{\nu L} - n_{eL}]$$

$$I_{3R} = \frac{1}{2}[n_{uR} - n_{dR} + n_{\nu R} - n_{eR}] \; ,$$

(5a)

where R, L denote left, right helicity components, we can rewrite Eq. (4c) by virtue of Eqs. (4b) and (3b) as

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B - L) \; .$$

(5b)

¿From these manipulations, and the requirement that the electric charge Q be conserved, we can draw two general conclusions about any preonic scheme obeying the charge,

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triality, and composite state assignments of Eqs. (1) and (2). From the first equality in Eq. (4c), we learn that at the preonic level, the electric charge must be constructed from the ungauged conserved total preon number F and a conserved gauged $U(1)$ charge proportional to $n_T - n_V$. From Eq. (5b), we learn that in an effective gauge theory of the composites, the electric charge necessarily has the form found in left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak models [8], which by a well-understood symmetry breaking mechanism can give the standard $SU(2)_L \times U(1)_Y$ electroweak model, and so we expect to make the connection with standard model physics by this route.

3. The role of the 10 and 15 representations of $SU(4)$

Since the generalized rishon model described in the previous section is a calculus of $SU(3)$ trialities, any embedding of $SU(3)$ in a larger gauge group can potentially give a preon model of this type. An extensive tabulation of color $SU(3)$ embeddings has been given in the review of Slansky [9], and there is clearly a plethora of possible models. To narrow the field, we introduce a ground rule derived from supersymmetry considerations: We will only consider gauge groups that are potentially embedable in an extended Poincaré super-multiplet. An examination of the spin 1 content of such multiplets [10] shows that the only possibilities admitting an $SU(3)$ embedding are the $SO(N)$ gauge groups with $N = 6, 7, 8$; we will choose as our candidate model the smallest of these, $SO(6) \sim SU(4)$, which has an adjoint multiplet of 15 gauge gluons and has long been considered [11] a possible unification group. For $N = 6$, extended supergravity requires that the fermions come in multiplets of either 6 or 20 two-component states; an examination of possible $SU(4)$ representations corresponding to these numbers shows that the only case leading to a satisfactory representation of the triality and charge rules of Eq. (1) corresponds to putting the fermions in left

handed 10 and $\overline{10}$ representations of $SU(4)$. ¿From the viewpoint of a possible supergravity extension it is very encouraging that, as discussed below in Sec. 4, these representations correspond *precisely* to the decomposition of the antisymmetric tensor representations 15 and 20 of $SU(6)$, the automorphism group used [12] in constructing the $N = 6$ extended Poincaré multiplet, under the irregular embedding of the $SU(4)$ subgroup.

The specific properties of $SU(4)$ that are needed for our model are given in Tables 25-27 of Slansky [9], and some related properties of $SU(3)$ that we use are given in Tables 23 and 24. As motivated in the preceding paragraph, our model consists of a 10_L and a $\overline{10}_L$ of two-component Weyl spinor preons (or equivalently, of a single 10 of Dirac four-component fermionic preons) gauged by the 15 adjoint representation of $SU(4)$. Since our model is vectorlike, there are no gauge anomalies and the model is renormalizable. Because the 10 representation has quadrality 2, the $\overline{10}$ has quadrality $-2 \equiv 2$ modulo 4, and so any odd number of preons has quadrality 2 modulo 4. Thus fermionic composites can never be $SU(4)$ singlets, and so the hypercolor forces in the fermionic sector of the model are frustrated [13].

We postulate that as a consequence, the $SU(4)$ symmetry is spontaneously broken into the maximal $SU(3) \times U(1)$ subgroup by a condensate, to be discussed further in Sec. 4, characterized by an energy scale *that is much smaller* than the characteristic scale Λ_H at which the SU_4 running coupling becomes strong. Even at the scale Λ_H, a very small asymmetric perturbation is enough to specify the favored subgroup into which the $SU(4)$ symmetry breaks, and moreover, when the running coupling becomes large, small asymmetries can be amplified and have a decisive effect on the dynamics. Thus our operating assumption will be that at the scale Λ_H where the $SU(4)$ dynamics becomes strongly coupled, the theory reorganizes itself according to the $SU(3)$ content of the $SU(4)$ multiplets.

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More specifically, we shall assume that *in each sector characterized by definite preon number and $SU(3)$ triality the fundamental fields with triality nonzero bind to form composites characterized by the smallest $SU(3)$ Casimir available for that triality.* Let us now explore the consequences of this assumption.

We begin with the gluon 15 multiplet, which under $SU(4) \supset SU(3) \times U(1)$ decomposes as

$$15 = 1(0) + 3(-1/3) + \overline{3}(1/3) + 8(0) \quad , \tag{6}$$

where the numbers in parentheses are the $U(1)$ charges, for which we adopt a normalization that is $1/4$ of that used in Slansky's Table 27. We see that the multiplet contains 9 gluons that are $U(1)$ neutral: the $U(1)$ force carrier $1(0)$ and the 8 $SU(3)$ force carriers $8(0)$. If we imagine all $SU(4)$ states displayed on a three dimensional plot with the $U(1)$ charge running along the z axis, then these 9 gluons lie in the xy plane, and we shall refer to them collectively as "horizontal gluons". The remaining 6 gluons of $SU(4)$ consist of two $SU(3)$ triplets 3 and $\overline{3}$, with $U(1)$ charges $\pm 1/3$; we shall refer to these collectively as the "vertical gluons".* According to our fundamental assumption, in the sector with $U(1)$ charge 1 and angular momentum 1, three vertical gluons in the $\overline{3}$ representation will bind, picking up horizontal gluon components of the wave function as needed, to form an $SU(3)$ singlet composite vector meson state with $U(1)$ charge 1; similarly, the three 3 gluons will bind to form a vector state with $U(1)$ charge -1. These composite vector states will ultimately play the role of components of the intermediate vector bosons $W^{\pm}$. Thus, our picture is that the

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* The familiar plot of the $SU(4)$ pseudoscalar meson 16-plet [14] takes this form, when the π, η, K mesons are relabeled "horizontal gluons" and the D mesons are relabeled "vertical gluons".

adjoint representation of the $SU(4)$ group "folds" in a manner dictated by the decomposition into $SU(3) \times U(1)$, with horizontal gluons remaining as massless gauge gluons, but with the vertical gluon triplets binding to form two color $SU(3)$ singlets that will become the charged weak force carriers.

We turn next to the 10_L and $\overline{10}_L$ representations, which under $SU(4) \supset SU(3) \times U(1)$ decompose as

$$10_L = 1_L(1/2) + 3_L(1/6) + 6_L(-1/6)$$

$$\overline{10}_L = 1_L(-1/2) + \overline{3}_L(-1/6) + \overline{6}_L(1/6) \quad ,$$

(7)

where the $U(1)$ charge normalization is again $1/4$ of that used by Slansky. Since the $SU(3)$ 6 representation has triality $2 \equiv -1$ modulo 3, if we assign states S_L, T_L, and V_L to the $\overline{10}$ of $SU(4)$, and their antiparticles correspondingly to the 10, according to

$$S_L \equiv 1_L(-1/2) \ , \quad T_L \equiv \overline{6}_L(1/6) \ , \quad V_L \equiv \overline{3}_L(-1/6)$$

$$\overline{S}_L \equiv 1_L(1/2) \ , \quad \overline{T}_L \equiv 6_L(-1/6) \ , \quad \overline{V}_L \equiv 3_L(1/6) \quad ,$$

(8)

then the T and V states have the trialities required by Eq. (1). Furthermore, since the ungauged preon number $n_{10} = n_{10_L} - n_{\overline{10}_L}$ and the $U(1)$ charge $Q_{U(1)}$ are both conserved, a satisfactory definition of the conserved electric charge operator Q at the preonic level is

$$Q = -\frac{1}{6}n_{10} + Q_{U(1)} \quad ,$$

(9)

according to which the charges of S, T, and V are

$$Q_S = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3} \ , \quad Q_T = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \ , \quad Q_V = \frac{1}{6} - \frac{1}{6} = 0 \quad ,$$

(10)

in agreement (for T and V) with the charge assignments of Eq. (1). Therefore the T and V preonic states in the 10 and $\overline{10}$ give a realization of the "rishon" model. According to our fundamental assumption, in the three preon angular momentum $1/2$ sector the T's and V's will bind, picking up horizontal gluons as needed,

into the states with lowest $SU(3)$ Casimir for each triality (the S's are color singlets and so do not directly participate), giving color triplet quarks and color singlet leptons according to the scheme of Eq. (2).

In addition to the three preon composites of Eq. (2), one in principle can have fractionally charged triality zero composites consisting of two preons and one antipreon, or of one preon and two antipreons, such as $TT\overline{V}$ and $T\overline{V}\overline{V}$. These states are discussed in Appendix B, where it is argued from the anomaly structure of the theory that they cannot simultaneously be present, along with the composites of Eq. (2), in a gauged electroweak low energy effective action.

4. Chiral symmetry structure of the model

We turn next to the crucial issue of the chiral symmetry structure of the model, and its implications for the spectrum of massless particles. According to what, until recently, was standard lore about chiral symmetry breaking based on perturbative studies supplemented by instanton [15] and lattice gauge theory arguments [16], a hypercolor gauge theory containing fermions with vector-like couplings would be expected to follow the "most attractive channel" [17] rule obeyed in QCD. According to this rule, one would expect a fermion-antifermion condensate (a Z_2 condensate in the terminology used below) to form, giving the composites masses at the hypercolor scale, and thus precluding their identification with standard model quarks and leptons. However, as noted in Sec. 1, recent results of Seiberg [3] (for expository reviews see Seiberg [4] and Intriligator and Seiberg [4]) show that by using holomorphy information in supersymmetric theories, one can find examples that contradict the most attractive channel rule. Specifically, these authors study supersymmetric QCD with N_f quarks in the fundamental and antiquarks in the anti-fundamental representation, which is

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a model in which all fermions have vectorlike couplings. This theory is asymptotically free for $N_f < 3N_c$. For $N_f = N_c$ the theory confines and breaks chiral symmetry, as expected from the most attractive channel rule, but for $N_f = N_c + 1$ (see Sec. 4.3 of Intriligator and Seiberg [4]) the theory exhibits confinement into composites without chiral symmetry breaking, contradicting the conclusion one would get from the most attractive channel rule. We interpret this example as indicating that the older standard lore about chiral symmetry breaking must be treated skeptically when dealing with supersymmetric theories.

Although the model of this paper, in a non-supersymmetric context, might be expected to follow the most attractive channel rule, as we have discussed above the model is motivated by the possibility of an $N = 6$ supersymmetric embedding. In the context of such an embedding, we take the results of [3, 4] to indicate that we contradict no known results in quantum field theory by postulating that chiral symmetry breaking in our model does not follow the most attractive channel rule, and that a Z_2 condensate does not form at the hypercolor scale. Instead, we assume that the only chiral symmetry breaking relevant at the hypercolor scale is that required by the instanton induced effective potential, which we now proceed to analyze, with further breaking of chiral symmetry occurring only at energies much below the hypercolor scale.

Because the fermion representation structure in our model is simply $10_L + \overline{10}_L$, there are only two global symmetry currents. The first,

$$V_\mu = \overline{\psi}_{10_L} \gamma_\mu \psi_{10_L} - \overline{\psi}_{\overline{10}_L} \gamma_\mu \psi_{\overline{10}_L} \quad , \tag{11a}$$

is the conserved vector preon number current, whose conserved charge is n_{10}. The second,

$$X_\mu = \overline{\psi}_{10_L} \gamma_\mu \psi_{10_L} + \overline{\psi}_{\overline{10}_L} \gamma_\mu \psi_{\overline{10}_L} \quad , \tag{11b}$$

is an axial current, the conservation of which is broken by the $U(1)$ axial anomaly. The effect of $SU(4)$ instantons, combined with the $U(1)$ anomaly, is to induce an effective chiral symmetry breaking potential which has the structure

$$\Delta V \sim \text{constant} \times [\psi_{10_L} \psi_{\overline{10}_L}]^6 \quad , \tag{12}$$

with $SU(4)$ and Lorentz indices contracted to form singlets (the details of this do not concern us), and with the exponent in Eq. (12) taking the value 6 because this is the index $\ell(10)$ of the 10 representation given in the Slansky [9] tables, which is one half the number of fermion zero modes [15] in an instanton background.* Under a chiral transformation of the left-handed fields,

$$\psi_{10_L} \to \psi_{10_L} \exp(i\alpha)$$
$$\psi_{\overline{10}_L} \to \psi_{\overline{10}_L} \exp(i\alpha) \quad , \tag{13a}$$

the symmetry breaking potential of Eq. (12) transforms as

$$\Delta V \to \Delta V \exp(12i\alpha) \quad , \tag{13b}$$

and so breaks the continuous chiral symmetry of the Lagrangian down to a discrete Z_{12} subgroup

$$\alpha = \frac{2\pi k}{12} \quad , \tag{13c}$$

with k any integer.

We now follow a remark of Weinberg [18] that a Z_K chiral symmetry can protect certain composites from acquiring masses, together with a suggestion of Harari and Seiberg

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* Equation (4.31) of Peskin [15] writes the exponent of Eq. (12) as $k = n_{10}C(10) + n_{\overline{10}}C(\overline{10})$, with Peskin's $n_{10} = n_{\overline{10}} = 1$ the number of Weyl spinors, and with Peskin's $C(10) = C(\overline{10}) = 3 = \ell(10)/2$ one half of the Slansky index $\ell(10)$.

[19], that a Z_K chiral symmetry can be used as a quantum number to distinguish between different quark lepton families. Because there are no conserved global axial vector currents in the model, there are no 't Hooft anomaly matching conditions to be satisfied, and so this Z_K analysis, with $K = 12$ in our case, is the sole criterion governing the appearance of low mass composite fermion states. We begin by noting that any fermionic composite is constructed from an odd number of fields ψ_{10}, $\psi_{\overline{10}}$ or their adjoints; therefore (since, if the sum of two integers is odd, so is their difference) the general composite monomial, which we denote by the Dirac spinor Ψ_N, transforms with an odd power $2N + 1$ under the Z_{12} chiral transformation of Eqs. (13a-c),

$$\Psi_N \rightarrow \exp[i2\pi\gamma_5(2N+1)k/12]\Psi_N \quad , \tag{14a}$$

which, since γ_5 has eigenvalues $1, -1$ on L, R chiral states, can be written as

$$\Psi_{N\,L,R} \rightarrow \exp[i2\pi(1,-1)(2N+1)k/12]\Psi_{N\,L,R} \quad . \tag{14b}$$

Let us now analyze the circumstances under which (i) two different monomials Ψ_N, Ψ_M, $N \neq M$ behave as members of distinct families, and (ii) the monomial Ψ_N is protected from acquiring a mass.

Assuming that the low energy effective theory remains a gauge theory, two different monomials Ψ_N and Ψ_M will represent distinct families if the chiral transformation of Eqs. (14a, b) forbids the occurrence of off-diagonal kinetic energy and gauge coupling terms

$$\Psi_M^\dagger \gamma_0\gamma_\mu \partial^\mu \Psi_N \ , \quad \Psi_M^\dagger \gamma_0\gamma_\mu \Psi_N A^\mu \quad , \tag{15a}$$

with A^μ a chirally invariant gauge potential. Thus the occurrence of the couplings of

Eq. (15a) will be forbidden if

$$[2N + 1 - (2M + 1)]k/12 = (N - M)k/6 \qquad (15b)$$

takes a noninteger value for any integer k. Clearly, for $N - M = 1, 2, 3, 4, 5$, Eq. (15b) is not an integer for $k = 1$, while when $N - M = 6P$ with P an integer, Eq. (15b) is integer-valued for all k. Hence the Z_{12} invariance of our theory implies that there are exactly 6 fermion families. Let us consider next what happens when the Z_{12} invariance group is successively broken by condensates to the subgroups Z_6, Z_4, and Z_2. For Z_6, the 6 in Eq. (15b) is replaced by 3, and the same reasoning implies that there are 3 fermion families, labeled by $N = 0, 1, 2$, the first family corresponding to a mixture of $N = 0, 3$ of the original 6, the second to a mixture of $N = 1, 4$ and the third to a mixture of $N = 2, 5$. When Z_{12} is broken to Z_4, the 6 in Eq. (15b) is replaced by 2, and our reasoning implies that there are 2 fermion families, labeled by $N = 0, 1$, corresponding to respective mixtures of the even and of the odd N values in the original 6. Finally, when Z_{12} is broken to Z_2, the 6 in Eq. (15b) is replaced by 1, and only 1 distinct family remains, in other words, all of the original 6 values of N are allowed to mix.

Now that we have determined the family structure, let us analyze when the families are required by the discrete chiral symmetry to remain massless. In carrying out this analysis, we will make the technical assumption that we only have to consider mass terms within each distinct family as characterized by the kinetic energy and gauge term analysis just given, and that we can ignore possible mass terms linking families that cannot couple through the kinetic and gauge coupling Lagrangian terms of Eq. (15a). As discussed in Appendix C, this assumption is equivalent to the assumption that the Coleman-Mandula [20] analysis of symmetries of the S matrix extends to the case in which the internal symmetry is a discrete

chiral symmetry as in Eq. (13), rather than a continuous symmetry. With this assumption, the monomial Ψ_N in general can acquire a Dirac mass if

$$m_D \Psi_{NR}^{\dagger} \gamma_0 \Psi_{NL} + \text{adjoint} \qquad (16a)$$

is invariant under the chiral transformation of Eqs. (14a, b), and (if neutral) it can acquire a Majorana mass [8] if

$$m_L \Psi_{NL}^{T} C^{-1} \Psi_{NL} + m_R \Psi_{NR}^{T} C^{-1} \Psi_{NR} + \text{adjoint} \quad , \qquad (16b)$$

with C the charge conjugation matrix, is similarly invariant. (In the above equations, m_D, m_R, m_L denote arbitrary complex constants.) Therefore, Ψ_N remains massless only if

$$2(2N+1)k/12 = (2N+1)k/6 \qquad (16c)$$

is not an integer for some integer k. Clearly, to analyze this condition we need only keep the residue of N modulo 3, so that we have the three cases $N = 0, 1, 2$ to consider. For $N = 0$ we have $2N + 1 = 1$ and for $N = 2$ we have $2N + 1 = 5 \equiv -1$ modulo 6, and so in both of these cases Eq. (16c) is noninteger as long as k is not divisible by 6. For $N = 1$, Eq. (16c) reduces to $k/2$ and is noninteger for odd k, and integer for even k. Hence in all cases the composite Ψ_N is required to remain massless by the Z_{12} invariance.

We next consider what happens to the mass analysis when Z_{12} is broken into one of its subgroups. When Z_{12} is broken to Z_6, the 6 in Eq. (16c) is replaced by 3. In this case, for $N = 0$ we still have $2N + 1 = 1$, and for $N = 2$ we have $2N + 1 = 5 \equiv -1$ modulo 3, and so in both of these cases Eq. (16c) remains a noninteger as long as k is not divisible by 3, and the corresponding Ψ_N is required to remain massless. On the other hand, when $N = 1$ we have $(2N+1)k/3 = k$, which is always an integer, so the $N = 2$ family is no longer protected

by discrete chiral invariance from acquiring a mass. Thus of the three distinct families left when Z_{12} is broken to Z_6, two remain massless, but the composites in the remaining one can acquire masses. When Z_{12} is broken to Z_4, the 6 in Eq. (16c) is replaced by 2. We now have $(2N+1)k/2 = Nk + k/2$, which is always nonintegral for odd k irrespective of the value of N. Hence the two distinct families left when Z_{12} is broken to Z_4 remain massless. Finally, when Z_{12} is broken to Z_2, the 6 in Eq. (16c) is replaced by 1, giving $(2N+1)k$ which is always an integer. The composites in the one remaining distinct family can then acquire masses.

To summarize, we have shown that a symmetry breaking chain

$$Z_{12} \rightarrow Z_6 \rightarrow Z_2 \tag{17a}$$

leads to a family mixing and mass generation chain

$$0, 1, 2, 3, 4, 5 \rightarrow (03), (25), (14)_M \rightarrow (012345)_M \quad , \tag{17b}$$

while a symmetry breaking chain

$$Z_{12} \rightarrow Z_4 \rightarrow Z_2 \tag{17c}$$

leads to a family mixing and mass generation chain

$$0, 1, 2, 3, 4, 5 \rightarrow (024), (135) \rightarrow (012345)_M \quad , \tag{17d}$$

where the numbers correspond to the N values of the original 6 massless families enforced by Z_{12}, where families with N values in parentheses

mix, and where a subscript M indicates that there is no discrete chiral protection against acquiring a mass. We shall show in the next section that in our model, Lorentz scalar,

electrically neutral, color singlet condensates exist corresponding to each step in the chains of Eqs. (17a-d). Thus our model is capable of generating the kind of family structure observed in the standard model: At the Z_6 level, there are three distinct families, two of which are protected from acquiring mass; at the Z_4 level two massless families remain, but they mix with the third heavy family; while at the Z_2 level the three Z_6 or two Z_4 families mix to form one family, all states of which can acquire mass. We shall leave to a future investigation a detailed study of the mass and mixing matrices that arise from this scheme. However, even without a detailed analysis, it is clear that for the model to be phenomenologically viable, the scale corresponding to the breaking $Z_{12} \to Z_6$ must be in the electroweak range 200 GeV to 1 TeV, and *our model then makes the prediction that there should be heavy duplicates of each fermion family in this range.* These are not expected to be mirror fermions, but rather should have weak couplings of the same type as their currently observed partners. Once we have associated the Z_6 condensate with the electroweak scale, our model implies on general grounds that breaking to Z_2 *must* occur at the QCD scale, because the 't Hooft anomaly matching conditions for quark binding into color singlets are not obeyed when more than one family is initially present [21]. (Of course, detailed QCD studies indicate that a chiral symmetry breaking Z_2 condensate appears even when only one family is present, although not required [21] by the anomaly matching conditions, which are satisfied for one family.)

To conclude this section, let us address the wider question of what groups and fermion representations could permit an analogous chain leading to 3 families. Clearly, for a representation with index ℓ, the analog of Eq. (15b) implies that the number of initially massless distinct families is ℓ. Our analysis shows that it is not possible to get 3 massless families without heavy partners, since when $\ell = 3$ we found that only 2 of the 3 families were protected from

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acquiring masses. In fact, among all the Lie groups catalogued by Slansky [9], only the group $SU(5)$ has a representation, the 10, with index 3. Under $SU(5) \supset SU(3) \times SU(2) \times U(1)$, the 10 decomposes as

$$10 = (1,1)(6) + (3,2)(1) + (\overline{3},1)(-4) \quad , \tag{18a}$$

and so if we define the charge as

$$Q = \frac{1}{15}[-n_{10} + Q_{U(1)}] \quad , \tag{18b}$$

we get two representations with the charge and helicity assignments assumed in Sec. 2. However, there are problems – since five 10's can bind to form a hypercolor singlet, there is no reason for non-singlet composites to dominate, and also, the V is an $SU(2)$ doublet while the T is an $SU(2)$ singlet. Nonetheless, this model deserves further study.

Turning to groups catalogued by Slansky with representations with index 6, there are the 8 of $SU(3)$, the 10 of $SU(4)$, the 20 of $SU(6)$, and the 28 of $SU(8)$. The first of these does not have the triality structure needed in Sec. 2; the second is the model of this paper; and the third is related to the model of this paper as follows. The sextality of the 20 of $SU(6)$ is 3, and so in a model

based on this representation, the $SU(6)$ forces are also frustrated in the fermionic sector. Under the irregular embedding $SU(6) \supset SU(4)$, the 20 of $SU(6)$ decomposes as [22]

$$20[SU(6)] = 10[SU(4)] + \overline{10}[SU(4)] \quad , \tag{19}$$

and the 15 of $SU(6)$ decomposes as

$$15[SU(6)] = 15[SU(4)] \quad . \tag{20}$$

This indicates that our model can also be obtained from $SU(6)$, a fact that is relevant for possible supergravity embeddings. Finally, we consider the 28 of $SU(8)$. Under $SU(8) \supset SU(3) \times SU(5) \times U(1)$, this representation decomposes as $28 = (1, 10)(6) + (3, 5)(-2) + (\bar{3}, 1)(-10)$, so we again encounter the problem that the 3 and the $\bar{3}$ of $SU(3)$ transform under different representations of the other non-Abelian factor group $[SU(5)$ in this case] in the decomposition.

One could also consider groups with representations with index ℓ larger than 6 but still divisible by 3. These include the 27 of $SO(7)$ with index 18, the 28 of $SO(8)$ with index 12, the 54 of $SO(10)$ with index 24, the 52 of F_4 with index 18, the 78 of E_6 with index 24, the 133 of E_7 with index 36, and the 248 of E_8 with index 60. However, for $\ell = 3k$ massless families with $k > 2$, the number of fermions becomes so large that $SU(3)$ is no longer asymptotically free. Hence models based on representations with $\ell > 6$ require an intermediate breaking to $SU(n)$ with $n > 3$, followed by a further breaking to $SU(3)$ accompanied by a reduction in the number of families.

5. The electroweak sector

We turn next to an examination of how electroweak forces acting on the composite quarks and leptons arise in our model. Far below the $SU(4)$ scale Λ_H, the gauge gluons that are present are the 9 horizontal gluons that mediate color $SU(3)$ and $U(1)$ forces and, we have argued, color singlet composites of the vertical gluons. We begin by showing that in the absence of a condensate breaking Z_{12} to Z_6, the vertical composites *cannot* account for the weak interactions. To see this, consider a transition from a d quark to a u quark, which according to Eq. (2) is, in rishon terms, a transition $\overline{TVV} \to VTT$, and by Eqs. (3a), (3b),

(4b) and (9) obeys the selection rules

$$\Delta(n_T - n_V) = 3\Delta(B - L) = 6\Delta Q_{U(1)} = 0$$

$$\Delta(n_T + n_V) = \Delta F = -\Delta n_{10} = 6 \quad .$$

$$(21a)$$

On the other hand, absorption of the color singlet composite of three vertical gluons in the

$\overline{3}$ representation causes the transition $\overline{T}VV \rightarrow \overline{V}TT$, and obeys the selection rules

$$\Delta(n_T - n_V) = 3\Delta(B - L) = 6\Delta Q_{U(1)} = 6$$

$$\Delta(n_T + n_V) = \Delta F = -\Delta n_{10} = 0 \quad .$$

$$(21b)$$

So evidently the selection rules for the weak interactions and for the composite of three

vertical gluons are mismatched, and can be brought into correspondence only through the

action of a Lorentz scalar (or pseudoscalar) color $SU(3)$ singlet electrically neutral condensate

$\mathcal{C}$ obeying the selection rules,

$$\Delta(n_T - n_V) = 3\Delta(B - L) = 6\Delta Q_{U(1)} = -6$$

$$\Delta(n_T + n_V) = \Delta F = -\Delta n_{10} = 6 \quad ,$$

$$(22a)$$

which, acting in conjunction with the absorption of a composite of three vertical $\overline{3}$ represen-

tation gluons, converts the selection rules of Eq. (21b) to those of Eq. (21a). Similarly, the

absorption of a composite of three vertical 3 representation gluons requires the action of the

conjugate condensate $\overline{\mathcal{C}}$ obeying the corresponding selection rules,

$$\Delta(n_T - n_V) = 3\Delta(B - L) = 6\Delta Q_{U(1)} = 6$$

$$\Delta(n_T + n_V) = \Delta F = -\Delta n_{10} = -6 \quad ,$$

$$(22b)$$

to produce the weak interaction transition $u \rightarrow d$.

Referring to the assignments of the $SU(3)$ components of the $\overline{10}_L$ state given in

Eq. (8), the following two possible condensates have the quantum numbers of Eq. (22a),

$$S_L^3 T_L^3 , \quad V_L^6 \quad ,$$

$$(23)$$

21

in both of which color and Lorentz indices are understood to be contracted to form, respectively, an $SU(3)$ singlet and a Lorentz scalar (or pseudoscalar). The following argument suggests that a condensate should indeed form in the $S_L^3 T_L^3$ sector. Consider first a pair $S_L T_L$, for which there is no $SU(3)$ binding force (the S is a singlet) and for which the product of $U(1)$ charges in Eq. (8), as well as of electric charges in Eq. (9), is negative, so that the Abelian binding force is attractive on all length scales. Since the interactions are attractive, these particles can form a loosely bound Lorentz scalar, color sextet, electrically neutral pair, and three such pairs can then bind by the $SU(3)$ color force to form a color singlet state. Alternatively, we can argue that the three sextet T_L's will bind to form a color singlet "nucleus" of $U(1)$ charge 1/2 and electric charge 1, that will then bind to three orbiting S_L's each of $U(1)$ charge $-1/2$ and electric charge $-1/3$, to make an electrically neutral composite. This argument also suggests that a V_L^6 condensate is unlikely, since the Abelian forces in this case are always repulsive or zero, and since V_L^3 can form a color singlet and the long range color force between two V_L^3 singlets vanishes. Even in the absence of a V_L^3 condensate, the $S_L^3 T_L^3$ condensate, which has the same quantum numbers, can cause effects such as neutrino-antineutrino mixing.

To sum up the discussion thus far, we have argued that an explanation of the weak interactions requires, and the force and multiplet structure of our model makes it likely that there exists, a condensate of sixth degree in the fields of the 10 representation. This condensate breaks the initial Z_{12} discrete chiral symmetry to Z_6, and thus has exactly the structure needed in the analysis of Sec. 4 to convert the initial six massless families to three families. In its role in allowing the exchange of vertical gluon composites to cause weak interaction transitions, the S state functions as an analog of the "doorway states"

[23] of nuclear physics. As already discussed in the preceding section, we assume that the energy scale characterizing the Z_6 condensate is much smaller than the hypercolor scale at which preon binding occurs. Since the strong interactions play a role in the formation of the condensate, it is in fact reasonable to expect that the condensate scale, which helps set the electroweak scale in our model, should lie closer to the $SU(3)$ scale Λ_{QCD} than to the hypercolor scale Λ_H. A discussion of the relationship between Λ_{QCD} and Λ_H is given in Sec. 6.

Let us now determine the algebraic structure of the resulting weak interactions, making throughout the assumption that the minimal algebra required by the quantum numbers is the one that occurs. Because the composites of three $\bar{3}$ or three 3 vertical gluons have electric charge $Q = \pm 1$, their action on the fermionic composites takes place only within the doublet pairs u, d or e, ν differing by one unit of electric charge. Hence the minimal algebraic structure containing their gauge charges is an $SU(2)$ of operators $W_+ = W_1 + iW_2, W_- = W_1 - iW_2, W_3$, with W_+ the gauge charge associated with the composite formed from three $\bar{3}$ vertical gluons, with W_- the gauge charge associated with the composite formed from three 3 vertical gluons, and with

$$[W_+, W_-] = 2W_3$$

$$[W_3, W_\pm] = \pm W_\pm \quad .$$

(24)

In Appendix A, we give an explicit calculation based on the $SU(4)$ gauge algebra, using the role of the S as a doorway state, that shows that the algebra induced by the action of the vertical gluon composites, irrespective of the form of their $SU(3)$ wave function, has precisely the form of Eq. (24). Let us now take into account the effect on this algebra of the presence of a broken Z_{12} symmetry. If we imagine switching off the condensate, the charges associated with the vertical gluons still obey Eq. (24), although they no longer cause weak

interaction transitions. But now we must take into account the fact that vertical gluons can bind with preon pairs, and hence we can get bosonic composites with differing behaviors under the Z_{12} chiral transformation of Eq. (14). Because a real boson field $W_{1,2,3}$ must be self adjoint, it cannot transform under Z_{12} with a complex phase; thus the only possibility allowed is that the vertical gluon composites pick up 6 preons in a charge zero, color singlet, Lorentz scalar state, producing a factor of -1 under the Z_{12} transformation of Eq. (14). Hence before the condensate is turned on, we must have two distinct families $W_A^{e,o}$ of $SU(2)$ charges associated with three vertical gluon exchange, with the following behavior under the Z_{12} transformations of Eq. (14),

$$
\begin{aligned}
W_A^e &\to W_A^e \\
W_A^o &\to W_A^o (-1)^k \quad ,
\end{aligned}
\tag{25}
$$

which implies that both sets of charges are invariant under the Z_6 chiral subgroup that survives when the condensate is turned on. Identifying the charges W^e with the W's introduced above Eq. (24), the minimal algebra with an $SU(2)$ structure in which Eqs. (24) and (25) are both satisfied is evidently

$$
\begin{aligned}
[W_A^e, W_B^e] &= i \sum_C \epsilon_{ABC} W_C^e \\
[W_A^e, W_B^o] &= i \sum_C \epsilon_{ABC} W_C^o \\
[W_A^o, W_B^o] &= i \sum_C \epsilon_{ABC} W_C^e \quad ,
\end{aligned}
\tag{26a}
$$

which can be immediately diagonalized by forming the combinations $W_A^{e\pm o}$ to give

$$
\begin{aligned}
[W_A^{e+o}, W_B^{e+o}] &= i \sum_C \epsilon_{ABC} W_C^{e+o} \\
[W_A^{e-o}, W_B^{e-o}] &= i \sum_C \epsilon_{ABC} W_C^{e-o} \quad .
\end{aligned}
\tag{26b}
$$

The algebra of Eq. (26a) is isomorphic to the usual current algebra of vector and axial vector charges, and that of Eq. (26b) to the diagonalization of this algebra in terms of chiral

charges. Since the vector and axial vector charges give the *only* isomorphic images of the algebras of Eq. (26a,b) that can be formed from the composite fermions, the independent composite vector boson states $W_A^{e\pm o}$ *must* couple to the composite fermions with a chiral $SU(2)_L \times SU(2)_R$ gauge theory charge structure. Finally, since we have seen in Sec. 2 that the $U(1)$ horizontal gluon couples to the composite fermions through the charge $B - L$, the low energy effective gauge theory of the composite fermions has the charge structure*

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Note that this electroweak Lie group has rank 3, so that when the octet of horizontal gluons, which carry the color $SU(3)$ force, is included in the accounting, the low energy effective left-right symmetric theory has rank 5, that is, there is a set of 5 mutually commuting generators. At first it may seem paradoxical that a fundamental $SU(4)$ theory of rank 3 could give rise to a low energy effective theory of rank 5. Indeed, this would not be possible in a standard grand unification framework, where the low energy effective theory is always a subgroup of the full grand unification group, and so must have a rank not exceeding that of the unification group. However, in our model, only the $SU(3)_{\text{color}} \times U(1)_{B-L}$ subgroup of the effective theory, which has rank 3, is constructed from generators that are a subgroup of the

---

* The charge $Q_{U(1)}$ can only commute with an $SU(2)$ obeying the selection rule $\Delta Q_{U(1)} = 0$; hence Eq. (21a) shows that $U(1)_{B-L}$ commutes with the physical W states, which include the action of the condensate, whereas Eq. (21b) shows that $U(1)_{B-L}$ does not commute with the composite of three vertical gluons uncoupled to the condensate. We remark also that our argument for a chiral $SU(2)_L \times SU(2)_R$ structure based on a Z_{12}-induced doubling of the gauge algebra does not carry over to the vector-like $U(1)_{B-L}$ and color $SU(3)$ sectors, because the chiral currents which correspond to these are anomalous.

generator algebra of the unification group $SU(4)$, and this subgroup of the effective action obeys the usual rank restriction. The remaining $SU(2)_L \times SU(2)_R$ group is constructed entirely from generators that are *composites* of the fundamental generators, and this permits the rank of the Lie algebra characterizing the low energy effective action to be enlarged. As shown in Appendix A, compositeness allows the the construction (when the chiral doubling is taken into account) of two additional generators that commute with each other, with the generator of $U(1)_{B-L}$, and with the color isospin and color hypercharge generators, leading in all to five mutually commuting generators and a rank 5 effective Lie algebra.

Let us now make the *assumption* that this charge structure is implemented by the dynamics as a local gauging; this is clearly a step that will have to be justified in future work. We note, though, that recent results on supersymmetric models [3, 4] shows that there exist theories in which the low energy effective action contains composite local gauge degrees of freedom that differ qualitatively from those in the original Lagrangian. Generalizing from the example described in Sec. 5.4 of the lectures of Intriligator and Seiberg [4], one might infer that the rank of the dynamically generated gauge group should be smaller than the rank of the fundamental gauge group, a restriction that is satisfied by our proposal since the rank of $SU(2)_L \times SU(2)_R$ is less than the rank of $SU(4)$. (Of course, from a limited class of examples, many generalizations are clearly possible.) With the assumption of a local gauge dynamics, the low energy effective theory is an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory, which has a well-studied symmetry breaking pathway [8] leading to the $SU(2)_L \times U(1)_Y$ standard electroweak model. A variant of the usual symmetry breaking discussion will be needed in our case, because as we have seen in Eq. (22b) the condensate, which has nonzero $U(1)$ charge $|Q_{U(1)}| = 1$, already breaks the $U(1)$ gauge invariance.

We emphasize that the argument for the development of an $SU(2)_L \times SU(2)_R$ theory has not assumed parity violation, but only used the properties of the $Z_{12} \to Z_6$ symmetry breaking chain. If the condensate responsible for this symmetry breaking is scalar, then the left and right gauge couplings g_L and g_R will be equal, and the theory is left-right symmetric. Parity violation then arises from the breaking of the left-right symmetric theory to the standard model. An alternative possibility is that the condensate breaking the Z_{12} symmetry is P, and CP, violating. In this case the gauge couplings g_L and g_R will differ, and left-right symmetry is already violated at the level of the $SU(2)_L \times SU(2)_R$ theory. This variant is also known [24] to have a symmetry breaking route to the standard model.

In addition to the condensate $\mathcal{C}$ breaking Z_{12} to Z_6, it is also possible that there is a second charge zero condensate of the form $S_L T_L \overline{V}_L^2$, with Lorentz and color indices contracted to form a scalar and singlet respectively, that breaks Z_{12} to Z_4. The argument for three families in Sec. 4, and the analysis of electroweak structure in this section, can survive only if such an additional condensate is effectively a small perturbation on the condensate $\mathcal{C}$.

We note in concluding this section that there are many questions connected with the symmetry breaking mechanism in our model that need further detailed study. Among them are: (i) We have argued that the two triplets, totaling 6 vertical gluons, of the fundamental Lagrangian are replaced, in the effective Lagrangian acting on fermionic composites, by two triplets of $SU(2)$ gauge bosons, again totaling 6 gauge fields. Is this correspondence of numbers a coincidence, or does it have deeper significance? To answer this question, one will need a generalization of the standard analysis [25] of effective Lagrangians to the case in which the transformation between phenomenological field variables ϕ and fundamental field

variables χ has, instead of the regular form $\phi = \chi F[\chi]$, $F[0] = 1$, the singular form $\phi = \chi^3 F[\chi]$, $F[0] = 1$. (ii) We have only employed symmetry breaking mechanisms that break *discrete* global invariances, never continuous global invariances. Are there corresponding Goldstone bosons? Since the condensate breaks the local $SU(4)$ gauge invariance, and since the breaking of local gauge invariances is possible only after gauge fixing [26], it would appear that any associated Goldstone bosons should be gauge variant, and therefore may not be physical. Even if physical, by the analysis of [25] they will consist of a color triplet and antitriplet of scalars (analogous to the vertical gluons) corresponding to the coset manifold $SU(4)/SU(3)$, and by our binding postulate should bind to form color singlet composites. Can these composites, in analogy with conventional technicolor scenarios [8, 27], play a role in the electroweak Higgs sector? These are subtle issues that need further study. (iii) As we have already noted, the condensate has integer $U(1)$ charge and therefore partially breaks the original $U(1)$ gauge invariance, so that $Q_{U(1)}$ is only conserved modulo an integer and $B - L$ is only conserved modulo 2. At the same time, the condensate is electrically neutral and so does not break conservation of the electric charge Q. Thus our model is consistent with the emergence, after all symmetry breakings, of an unbroken electromagnetic $U(1)$ gauge field, but the detailed route by which this happens has to be established.

6. The renormalization group and the gauge hierarchy

According

to the picture developed in the preceding sections, the gauge forces associated with $SU(4)$ are responsible both for preonic binding at a high scale Λ_H, and for the electroweak forces and the strong QCD force at much lower scales Λ_{EW} and Λ_{QCD}. What can be said about the relation between these scales? An analysis of the relation between Λ_{EW} and

Λ_{QCD}, and of the value of the Weinberg angle, will require a detailed understanding of the dynamics of the condensate and its effect on the $U(1)$ couplings, and will not be attempted here. However, since the condensate is a color singlet it has no effect on the evolution of the $SU(3)$ coupling, allowing us to consider the relation between Λ_H and Λ_{QCD} as a simpler problem isolated from electroweak complications.

There are then three regimes to consider, as shown in Fig. 1, consisting of two branches of the running coupling, separated by a nonperturbative strong coupling regime in which a running coupling is insufficient to describe the dynamics.* For energies well above the scale Λ_H, branch 1 of the running coupling, describing the interaction of the preons, is given by the $SU(4)$ formula

$$g^2_{SU(4)}(\mu^2) = \frac{1}{b_{SU(4)} \log(\mu^2/\Lambda_H^2)} \quad , \tag{27a}$$

while for energies well below Λ_H and well above Λ_{QCD}, branch 2 of the running coupling, describing the residual color interactions of composite quarks, is given by the $SU(3)$ formula

$$g^2_{SU(3)}(\mu^2) = \frac{1}{b_{SU(3)} \log(\mu^2/\Lambda_{QCD}^2)} \quad . \tag{27b}$$

For energies in the vicinity of Λ_H, neither of these formulas applies, since the $SU(4)$ preonic coupling is strong, leading to the binding of both preons and vertical gluons on a length scale Λ_H^{-1}. Tracing the dynamics across this strong coupling nonperturbative region separating the two branches is a complicated but in principle calculable problem, the result of which will be

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* There are actually four regimes to be considered, since the running of the $SU(3)$ coupling speeds up below the scale where the three heavy partners of the quarks in the three families are frozen out; we shall assume that this scale is much closer to Λ_{QCD} than to Λ_H, and ignore this complication in the following discussion.

a "connection formula" fixing the relation between the scales Λ_H and Λ_{QCD} characterizing the two branches of the running coupling. The result of this calculation, we shall now show, can always be expressed as the value of the coupling g_*^2 at the point μ^2 where Eq. (27a) and the high energy extrapolation of Eq. (27b) intersect, i.e., where

$$g_*^2 \equiv g_{SU(4)}^2(\mu^2) = g_{SU(3)}^2(\mu^2) \quad . \tag{28}$$

Such an intersection always exists provided that $b_{SU(4)}$ is larger than $b_{SU(3)}$, in other words, provided the $SU(4)$ coupling is running faster than the $SU(3)$ coupling, since then the ratio

$$\frac{g_{SU(3)}^2(\mu^2)}{g_{SU(4)}^2(\mu^2)} \tag{29a}$$

is smaller than unity for μ^2 just above Λ_H but approaches the value

$$\frac{b_{SU(4)}}{b_{SU(3)}} \quad , \tag{29b}$$

which is larger than unity, as $\mu^2 \to \infty$. In terms of the coupling g_*^2 at the intersection, the relation between the scales Λ_H and Λ_{QCD} takes the form

$$\frac{\Lambda_H^2}{\Lambda_{QCD}^2} = \exp[(b_{SU(3)}^{-1} - b_{SU(4)}^{-1})/g_*^2] \quad . \tag{30}$$

The values of the renormalization group beta function coefficients $b_{SU(3),SU(4)}$ can be computed from the formula [9]

$$b = -\frac{\mu}{g^3}\frac{dg}{d\mu} = \frac{1}{32\pi^2}\left(\frac{11}{3}\ell(\text{vector}) - \frac{4}{3}\ell(\text{Dirac fermion}) - \frac{1}{6}\ell(\text{real spinless})\right) \quad , \tag{31}$$

where the ℓ values are the sums of the Slansky indices for the representations in which the particles lie.* To make a preliminary estimate, we defer the issue of dealing with possible

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* In Eq. (31) the familiar constant $16\pi^2$ has been replaced by $32\pi^2$, to compensate for

fundamental or composite scalars in our model, and thus simply ignore a possible spin zero contribution to Eq. (31). Then in the formula for $b_{SU(4)}$ we have $\ell(\text{vector}) = \ell(15) = 8$ and $\ell(\text{Dirac fermion}) = \ell(10) = 6$, while in the formula for $b_{SU(3)}$ we have $\ell(\text{vector}) = \ell(8) = 6$ and $\ell(\text{Dirac fermion}) = 12\ell(3) = 12$, where we have taken into account the fact that the preons bind to give 6 families, each containing 2 quark triplets. Substituting these values into Eq. (31) and then evaluating Eq. (30), we find

$$\frac{\Lambda_H^2}{\Lambda_{QCD}^2} = \exp[23\pi^2/(6g_*^2)] = \exp(37.8/g_*^2) \quad . \tag{32}$$

The relatively large exponent in Eq. (32) is a result of the small ratio $b_{SU(3)}/b_{SU(4)} = 9/32$, expressing the fact that the $SU(3)$ coupling runs much more slowly than the $SU(4)$ coupling as a result of the large number of composites created by the binding of the preons, and so in this respect the behavior of our model is reminiscent of the behavior of "walking technicolor" [27] theories. Equation (32) can give a large hierarchy ratio for values of g_* not much smaller than unity; for example,

for $g_* = 0.7$ one finds $\Lambda_H \sim 10^{16} \text{GeV}$, and hence a large hierarchy ratio is natural in our model.

For our model to be compatible with experimental limits on proton decay, it is crucial that the hierarchy ratio be very large, and that the vertical gluons, as well as the preons, be confined within a radius Λ_H^{-1}. Exchange of a single unbound vertical gluon can lead to the reaction $u + u \to \bar{d} + e^+$, and hence to proton decay.

With confined vertical gluons and preons, the reaction amplitude is proportional to

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the fact that Slansky's index ℓ is normalized to be always an integer, so that in Slansky's tables $\ell(\text{fundamental}) = 1$ and $\ell(\text{adjoint}) = 2N$ for the group $SU(N)$. Equation (31) agrees with the standard result $b = (16\pi^2)^{-1}(11 - 2N_f/3)$ for N_f flavor QCD.

the cross sectional area of a u quark, and hence the rate varies as Λ_H^{-4}. This dependence of the proton decay rate on the high mass scale is compatible [28] with experimental bounds, for values of Λ_H well below the Planck mass. As long as Λ_H is big enough for proton decay to be below current experimental limits, other effective four fermion (i.e., dimension 6) interactions resulting from the composite structure of quarks and leptons will automatically be much smaller than the current limits on such effective action terms.

7. Discussion: possible supergravity embeddings
and phenomenological implications

We turn finally to the issue of the embedding of our model in a larger structure, including gravitation, and to a brief discussion of phenomenological implications. We first remark that if our model is augmented by an $SU(4)$ 6 representation of Majorana fermions, the basic mechanisms discussed above can still work. The quadrality of the 6 is also 2, and so the $SU(4)$ forces remain frustrated in the fermionic sector. If the 6 representation develops its own Z_2 condensate at a high mass scale, it acts as an inert spectator in the Z_{12} chiral symmetry breaking chain for composites formed within the 10 representation, although it could make a contribution to the masses of these composites in the final stages of chiral symmetry breaking.

Let us now turn to consider gravity, or more specifically, supergravity. The existence of extended supergravity multiplets at one time briefly raised the hope that all matter fields, and gravitation, could be unified within one extended graviton multiplet. This hope could not be realized, however, in the framework of grand unification models or direct product hypercolor composite models, because the symmetry groups involved are too large to fit within the largest ($N = 8$) extended graviton multiplet [10]. An attractive feature of the model presented here is that this objection no longer applies: as already noted above, the $SU(4)$ symmetry group employed, and the required 15 spin one gauge gluons and 10 Dirac fermions (equivalent to 20 Majorana or Weyl fermions) correspond to representations appearing in the $N = 6$ supergraviton multiplet [10, 12] after reflection to insure CPT invariance. (The CPT invariant structure has an additional 6 Majorana fermions, that we noted are acceptable additions to the model.) Hence if the model given here proves viable, it becomes an

important question to construct a supergravity (and perhaps also a conformal supergravity [29]) field theory Lagrangian with the requisite gauge gluon and fermion representation structures. Such a theory would provide the possibility of a truly elegant unification of all of the forces. We note, finally, that any extended supergravity embedding of our model will contain fundamental scalars, with a representation structure that is dependent on the structural details of the extension. It is for this reason that we have not attempted at this stage an exploration of the scalar structure of our model; without solving the problem of classifying the possible supergravity embeddings, there appear to be many possibilities for both fundamental and composite scalars consistent with the vector and spinor dynamics that we have outlined.

The principal phenomenological implication of our model is that for each of the three standard model families of quarks and leptons, there is a corresponding heavy family with the same quantum numbers (*not* a mirror family), some members of which could have masses as low as those characterizing the third standard model family. We must first ask whether the presence of such extra families is compatible with experiment? Under the simplifying assumption that the effect of these heavy fermions on electroweak radiative corrections can be parameterized solely through the S parameter [30], the existence of 3 heavy families (6 heavy flavors) leads* to a contribution to S of $\frac{2}{\pi}$, which is only marginally compatible with the LEP data as reported in [30]. However, two caveats are relevant here. The first is that in our model the heavy states can mix with their standard model counterparts, and consequently [30] the implications of these states for electroweak physics cannot be fully parameterized within the standard S, T, U phenomenological framework. The second is that

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* I am indebted to R. N. Mohapatra for this observation.

scalar particles can lead to negative contributions to S (see, e.g., [30, 31]), and a supergravity embedding of our model will contain many scalars. Thus, electoweak radiative corrections do not at this

point make a decisive statement about the viability of our prediction of heavy family partners. We remark that in an $N = 6$ supersymmetry embedding of our model, these heavy partners could be the first signal for supersymmetry, contrasting

with the prediction of an "s" partner for each standard model particle by $N = 1$ supersymmetry extensions of the standard model.

Recently, the H1 and ZEUS collaborations at HERA have reported [32] an excess of positron-jet events at large Q^2, and so it is natural to ask if these events might signal the discovery of a heavy family partner E^+ of the positron e^+. As formulated up to this point, our model cannot account for the HERA events; the reason is that if color $SU(3)$ remains an exact symmetry, and if color neutralization is assumed to be instantaneous, the E^+ would be produced as a color singlet, and its dominant decay mode would be the electromagnetic decay $E^+ \rightarrow e^+ + \gamma$, which does not correspond to the e^+ plus jet signature reported by the HERA groups. However, the production and decay modes of the E^+ in our model depend strongly on the details of the surviving unbroken symmetries, and of color neutralization.

Suppose, as suggested by Slansky, Goldman, and Shaw [33], that color $SU(3)$ were weakly broken to "glow" $SO(3)$, while maintaining triality conservation modulo 3, with color gluons not in the $SO(3)$ subgroup acquiring very small masses, and with the threshold for excitation of free color dropping from infinity to a finite value of order 200-300 GeV. [It makes sense to talk about a free color threshold in this context because under $SU(3) \supset SO(3)$, all states of $SU(3)$ decompose into diality zero integer L representations of SO(3); half-

integer, nonzero diality representations of the covering group $SU(2)$, which are confined, are never encountered.] The possibility of the breaking of color to glow can naturally be incorporated into our model, for two reasons. First, the state classification of Secs. 3 and 4 uses only the conservation of triality modulo 3, but not the details of the particular $SU(3)$ representations corresponding to each triality sector, and so a triality preserving breaking of $SU(3)$ to $SO(3)$ leaves this classification, and in particular the distinction between leptons and quarks, intact. Second, to achieve a triality preserving breaking of $SU(3)$ to $SO(3)$, the smallest non-singlet $SU(3)$ representation with nonzero vacuum expectation must [33] be the 27, since this is the smallest triality zero representation of $SU(3)$ that contains an $SO(3)$ singlet. But since the postulated condensate $S_L^3 T_L^3$ of our model has an $SU(3)$ tensor product structure corresponding to the symmetric part of $6 \times 6 \times 6$, which contains the 27, it is consistent with the framework developed above to additionally postulate that the condensate has a small component in the 27 representation of color $SU(3)$ as well as a dominant color singlet component. The effect of a breaking of color $SU(3)$ to glow $SO(3)$, with a free color excitation threshold of order 200-300 GeV, would be to leave the standard model leptons as color singlets, but to permit their heavy counterparts to carry admixtures of color non-singlet states. Similarly, the heavy counterparts of the quarks would carry admixtures of triality ± 1 states other than the states $3, \overline{3}$, and electroweak anomaly cancellation would then give a sum rule relating the quark color state admixtures to those in the lepton states. In this scenario, the E^+ could carry a color octet component. It would be produced by positron gluon collision, and its dominant decay mode would be single color gluon emission $E^+ \to e^+ + g$, which would appear as a positron jet final state, corresponding to a "leptogluon" interpretation of the signature observed at HERA. As discussed in a recent phenomenological analysis of Akama,

Katasuura, and Terazawa [34], such an "excited positron" interpretation is consistent with the HERA data and with limits from other accelerator experiments.

An alternative scenario, which does not require color $SU(3)$ to be broken, is simply to assume that color neutralization does not fully take place in the hard processes involved in E^+ production and its subsequent decay. Recall that in our model, the e^+ and E^+ are both TTT three preon bound states, with an $SU(3)$ wave function (before color neutralization) corresponding to the mixed symmetry part of $6_L \times 6_L \times 6_R$, which has the Clebsch series $8 + 10 + \overline{10} + ...$ and contains no color singlet. Our postulate of Sec. 3 is that color neutralization occurs by picking up color gluons from the vacuum until the $SU(3)$ state with lowest Casimir is attained, so that only the $SU(3)$ triality plays a role in enumerating the possible states. However, in very hard processes, characterized by momentum transfers much larger than the QCD scale, it is possible that this color neutralization could be incomplete, and that the E^+ would then behave as a state with the color wave function suggested by the bare preon Clebsch series. Again, as in the color breaking scenario, this would permit the production of the E^+ by positron gluon collision, and its subsequent rapid decay into a positron and a gluon jet.

In summary, we suggest that the production and decay of the excess HERA events, interpreted as leptogluons, could be accounted for in our model when augmented by either the assumption that the Z_6 condensate that breaks $SU(4)$ to color $SU(3)$ contains a small component that further breaks color $SU(3)$ to glow $SO(3)$, or by the assumption that color symmetry remains exact but that color neutralization is incomplete in hard processes. On the other hand, a leptoquark interpretation of the HERA events is not apparent in our model; composite vector leptoquarks would be expected to have masses near Λ_H, since

there is no chiral or gauge symmetry argument for them to have small masses. Assessing the possibility of scalar leptoquarks will require further study of the related problems of supergravity embeddings and the scalar sector of our model.

Acknowledgments

Added note. After this manuscript was initially posted on the theory Bulletin Board, I received an interesting email from D. Fairlie and J. Nuyts who pointed out that the model given here, in which the fermions are in the 10 of $SU(4)$, fits into the general group theoretic framework of Fairlie, Nuyts and Taormina [35]. This paper (see its Appendix A) showed that a large class of preonic models constructed from fundamental preons must have exotic charge 1/6 fermion states, except in a rishon type model in which the rishons are composed of symmetrized pairs of preons (as in the 10 of $SU(4)$ used here, which is the symmetrical tensor product 4×4).

Appendix A.   The three vertical gluon composite gauge algebra

We compute here the gauge algebra corresponding to the action of a three vertical gluon composite on a preon bound state. Since the gluons couple to the preons through the representation matrices for the 10 representation of $SU(4)$, we must first construct these matrices. We do this by starting with the representation matrices Λ_A for the fundamental 4 representation (an explicit representation for them will be given shortly), and using the fact that since $4 \times 4 = 6 + 10$, the 10 representation is the symmetric part of the tensor product of two 4's. Let (ab) denote an index pair with $a, b = 1, ..., 4$, with the parentheses implying symmetrization, so that there are only 10 distinct values of (ab) when (ab) and (ba) are treated as equivalent. Then we can use (ab) as a label for the 16 rows and 16 columns of the representation matrices $M_{A(\bar{a}\bar{b})}^{(ab)}$ for the 10 representation. A simple computation using the group transformation law for a tensor product then gives

$$M_{A(\bar{a}\bar{b})}^{(ab)} = \frac{1}{2} \left(\Lambda_{A\bar{a}}^a \delta_{\bar{b}}^b + \Lambda_{A\bar{b}}^b \delta_{\bar{a}}^a + \Lambda_{A\bar{a}}^b \delta_{\bar{b}}^a + \Lambda_{A\bar{b}}^a \delta_{\bar{a}}^b \right) \quad , \tag{A1}$$

with $\delta_b^a = \delta_{ab}$ the Kronecker delta.

As discussed in the text, the 10 representation of $SU(4)$ contains a 1, a 3, and a 6 of $SU(3)$; let us order these so that the label (44) denotes the 1, the label $(4a)$, $a = 1, 2, 3$ denotes the 3, and the label (ab), $a, b = 1, 2, 3$ denotes the 6. We have seen that the weak interactions require the intervention of a condensate $S_L^3 T_L^3$, where $\overline{S}, \overline{V}, \overline{T}$ denote respectively the $SU(3)$ states $1, 3, 6$. Thus (ignoring the possibility of neutrino-antineutrino mixing) the only vertical gluon transitions between the preons relevant for the weak interactions are those between the $\overline{V}$ and the $\overline{S}$, described by the submatrix of Eq. (A1) with one index pair equal to (44) and one index pair equal to $(4a)$, $a = 1, 2, 3$, and with the index A corresponding to the triplet or antitriplet of vertical gluons.

At this point in the calculation it is convenient to introduce a specific representation for the $SU(4)$ fundamental representation matrices Λ_A. Let $\lambda_A, A = 1, ..., 8$ be the standard Gell-Mann matrices for $SU(3)$; then we take the first 8 $SU(4)$ matrices to be

$$\Lambda_A = \text{diag}(\lambda_A, 0) \quad , \tag{A2}$$

where we use the notation $\text{diag}(\alpha, \beta)$ to indicate a 4×4 block diagonal matrix with a 3×3 diagonal block α and a 1×1 diagonal block β. The remaining 7 $SU(4)$ matrices consist of the $U(1)$ generator

$$\Lambda_{15} = 6^{-1/2}\text{diag}(1, -3) \quad , \tag{A3}$$

and the six generators $\Lambda_{9,...,14}$ for the vertical gluons. It is convenient to write the latter in the form of an $SU(3)$ triplet and an $SU(3)$ antitriplet

of non-self-adjoint raising and lowering operators, $\tau_{\pm k}, \ k = 1, 2, 3$, defined by

$$(\tau_{+k})^a_{\bar{a}} = \delta^a_k \delta^4_{\bar{a}}$$
$$(\tau_{-k})^a_{\bar{a}} = \delta^a_4 \delta^k_{\bar{a}} \quad , \tag{A4}$$

which obey the algebra

$$\tau_{+k}\tau_{+\ell} = \tau_{-k}\tau_{-\ell} = 0$$

$$\tau_{+k}\tau_{-\ell} = \text{diag}(D_{k\ell}, 0) \tag{A5a}$$

$$\tau_{-k}\tau_{+\ell} = \text{diag}(0, \delta_{k\ell}) \quad ,$$

where $D_{k\ell}$ is the 3×3 matrix with matrix elements

$$(D_{k\ell})^a_{\bar{a}} = \delta^a_k \delta_{\bar{a}\ell} \quad , \tag{A5b}$$

together with

$$\tau_{+k}\text{diag}(0, 1) = \tau_{+k} \ , \quad \text{diag}(0, 1)\tau_{-k} = \tau_{-k} \quad . \tag{A5c}$$

If we now write the 10 representation matrices of Eq. (A1) for the 6 vertical gluons in corresponding non-self-adjoint raising and lowering operator form, we get

$$M_{\pm k(\bar a \bar b)}^{(ab)} = \frac{1}{2}\left(\tau_{\pm k \bar a}^a \delta_{\bar b}^b + \tau_{\pm k \bar b}^b \delta_{\bar a}^a + \tau_{\pm k \bar a}^b \delta_{\bar b}^a + \tau_{\pm k \bar b}^a \delta_{\bar a}^b\right) \quad . \tag{A6}$$

¿From this, we find for the submatrix acting on the $SU(3)$ 1 and 3 states $\overline{S}$ and $\overline{V}$,

$$M_{\pm k(4b)}^{(4a)} = M_{\pm k(44)}^{(44)} = 0$$

$$M_{+ k(4a)}^{(44)} = M_{- k(44)}^{(4a)} = 0$$

$$M_{+ k(44)}^{(4a)} = (\tau_{+ k})_4^a$$

$$M_{- k(4a)}^{(44)} = (\tau_{- k})_a^4 \quad , \tag{A7}$$

with $a, b = 1, 2, 3$ only. Thus this submatrix has the same structure as the corresponding 4 representation matrix $\tau_{\pm k}$ acting on a 4 of $SU(4)$ constructed from the $\overline{S}$ and $\overline{V}$ states, and therefore to study the algebra of the three vertical gluon composites as it acts in the $\overline{SV}$ subspace, it suffices to study this algebra using the 4 representation matrices of Eq. (A4).

This calculation is relatively straightforward. The charge operators $U_\pm$, which describe the action on 4 representation preon triples with charge matrices $\tau^{(1)}$, $\tau^{(2)}$, $\tau^{(3)}$, of the triality 0 composites formed from the triplet and antitriplet of vertical gluons, are

$$U_\pm = \sum_{kmn} C_{kmn}\tau_{\pm k}^{(1)}\tau_{\pm m}^{(2)}\tau_{\pm n}^{(3)} \quad , \tag{A8}$$

with C_{kmn} a tensor determined by the internal structure of the three gluon composite. From the algebraic properties of τ matrices in Eq. (A5a-c), we see that

$$U_+^2 = U_-^2 = 0$$

$$U_- U_+ = KP$$

$$U_+ P = U_+$$

$$PU_- = U_- \quad , \tag{A9a}$$

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with K the constant

$$K = \sum_{kmn} C_{kmn}^2 \quad , \tag{A9b}$$

and with P the projector

$$P = \text{diag}(0,1)^{(1)}\text{diag}(0,1)^{(2)}\text{diag}(0,1)^{(3)} \quad . \tag{A9c}$$

So defining charge operators $W_\pm$ and W_3 by

$$W_\pm = K^{-1/2}U_\pm \ , \quad W_3 = \frac{1}{2K}[U_+,U_-] \quad , \tag{A10a}$$

we see that

$$[W_+,W_-] = 2W_3$$

$$[W_3,W_+] = \frac{1}{2K^{3/2}}(U_+U_-U_+ - U_-U_+U_+ - U_+U_+U_- + U_+U_-U_+)$$

$$= \frac{1}{2K^{3/2}}2U_+U_-U_+ = K^{-1/2}U_+ = W_+ \tag{A10b}$$

$$[W_3,W_-] = \frac{1}{2K^{3/2}}(U_+U_-U_- - U_-U_+U_- - U_-U_+U_- + U_-U_-U_+)$$

$$= \frac{-1}{2K^{3/2}}2U_-U_+U_- = -K^{-1/2}U_- = -W_- \quad .$$

Thus the W charges obey the $SU(2)$ algebra of Eq. (24) of the text, irrespective of the detailed structure of the internal wave function C_{kmn}.

In the special case in which the tensor C_{kmn} is the $SU(3)$ structure constant f_{kmn}, the composites $U_\pm$ are color singlets and commute with the whole horizontal $SU(3)$ Lie algebra, without requiring the addition of horizontal gluons to achieve color neutralization. Therefore, the commutator $[U_+,U_-]$ commutes with the $SU(3)$ Lie algebra; since this commutator carries $U(1)$ charge zero it also commutes with the $U(1)$ generator, and so a rank 4 set of generators is given by the $U(1)$ generator, the third component of color isospin and the color hypercharge, and the commutator $[U_+,U_-]$. This is an explicit example showing that when

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composite structures in the group generators are allowed, one can get effective Lie algebras with *higher* rank than that of the fundamental Lie algebra from which they are formed. [Since the left-right symmetric theory $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is actually of rank 5, we need one more mutually commuting generator, and we argue in Eqs. (24-26) of the text that this comes from the breaking of the discrete Z_{12} chiral symmetry to Z_6.]

Appendix B. Mixed preon-antipreon fermionic states

As noted at the end of Sec. 3 of the text, in addition to the three preon fermionic composites of Eq. (2), our model also permits the fractionally charged, triality zero, mixed preon-antipreon fermionic states

$$\ell_U = TT\overline{V}$$
$$\overline{\ell}_D = T\overline{V}V \quad , \tag{B1}$$

with respective charges $Q = 2/3$ and $Q = 1/3$, together with their corresponding antiparticles. [In the Harari-Shupe scheme [6], the absence of these states is enforced through an ad hoc "no mixing"rule; in the Harari-Seiberg model [5], this rule is implemented through the hypercolor triality assignments of the rishons, which prevents the states of Eq. (B1) from being hypercolor singlets.] For these states, we readily find that the analogs of the counting relations of Eq. (3b) are

$$\frac{1}{3}(n_T - n_V) = n_{\ell_U} - n_{\ell_D}$$
$$\frac{1}{3}(n_T + n_V) = \frac{1}{3}(n_{\ell_U} + n_{\ell_D}) \quad . \tag{B2}$$

A transition $\ell_D \rightarrow \ell_U$ evidently obeys the selection rules

$$\Delta(n_T - n_V) = 6$$
$$\Delta(n_T + n_V) = 0 \quad , \tag{B3}$$

which agree with those of Eq. (21b) for a transition induced by a three vertical gluon composite *without* the action of the condensate $\mathcal{C}$.

Also, we see that for the states of Eq. (B1), the charge $n_T - n_V$, which acted (with the condensate) as the electroweak $U(1)$ for the composites of Eqs. (2) and (3b), now acts as the third component of an electroweak $SU(2)$. Conversely, for the states of Eq. (B1), the charge $(n_T + n_V)/3$, which acted as the third component of the electroweak $SU(2)$ for the composites of Eqs. (2) and (3b), now acts as an electroweak $U(1)$. Thus the electroweak interactions for the composites of Eq. (B1) have a structure incompatible with those for the standard model composites of Eq. (2). This point is further underscored if we add the helicity indices L, R to the electroweak groups and look at the corresponding anomaly structure. The ordinary family particles do not produce an anomaly in the $SU(2)$ current with chiral charge $n_T + n_V$, but do produce an anomaly in the $U(1)$ current with chiral charge $n_T - n_V$.

Conversely, the states of Eq. (B1) produce no anomaly in the $SU(2)$ current with chiral charge $n_T - n_V$, but do produce an anomaly in the $U(1)$ current with chiral charge $(n_T + n_V)/3$.

These facts mean that the low energy effective action of our model cannot simultaneously contain both the standard model particles of Eq. (2), and the fractionally charged particles of Eq. (B1), along with their electroweak gaugings. We interpret this to mean that our model can exist in two phases. In one phase, in which there is no $S_L^3 T_L^3$ condensate, the low energy spectrum consists of the fractionally charged particles of Eq. (B1) and their strong and electroweak gauge bosons.

In the second phase, which we are assuming to be the physically realized one, there is an $S_L^3 T_L^3$ condensate and the low energy spectrum consists of the standard model fermions of Eq. (2) and their strong and electroweak gauge bosons. In the assumed physical phase, there is a mechanism for giving the fractionally charged states of Eq. (B1) masses at the

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high scale Λ_H, since self energy graphs in which these states emit a three vertical gluon composite with chiral charge $n_T - n_V$ (without intervention of the condensate) will receive a divergent contribution from back to back insertions on the composite gluon propagator of anomalous triangle graphs containing the standard model fermions. Thus, according to the interpretation suggested here, the discussion of Secs. 4 and 5 of the text extends to show that the "no mixing" rule also emerges from the chiral symmetry structure of our model.

Appendix C. Does the Coleman-Mandula Theorem

Apply to Discrete Chiral Symmetries?

In the chiral symmetry analysis of Sec. 4, we made the technical assumption that we only had to consider mass terms *within* groups of states that are allowed to couple through the kinetic energy, and hence are classified as members of the same family. To see why this assumption is needed, consider the inter-family off-diagonal mass term

$$\Psi^\dagger_{MR}\gamma_0\Psi_{NL} + \text{ adjoint} \quad , \tag{C1a}$$

which is invariant under the Z_{12} chiral transformation of Eqs. (14a, b) when

$$(2M + 1 + 2N + 1)k/12 = (M + N + 1)k/6 \tag{C1b}$$

is an integer for all k, a condition that is satisfied for the combinations $(M, N) = (0,5)$, $(1,4)$, $(2,3)$. Similarly, a discrete Z_6 chiral invariance allows off-diagonal mass couplings between monomial pairs $(M, N) = (0, 2), (1, 1)$. In either of these cases, diagonalizing the mass term leads to mass eigenstates that are not discrete chiral symmetry eigenstates,

because the kinetic energy term for each mass eigenstate separately is not a discrete chiral invariant. Such structures, if present in the low energy effective action, would violate

the assertion of the Coleman-Mandula [20] theorem that the only symmetries of S-matrix are the direct product of the Poincaré group and an internal symmetry group. So we are justified in excluding them *if* the Coleman-Mandula theorem extends to the case of discrete internal symmetry groups, such as the discrete chiral transformations of Eqs. (13, 14).

An examination of the existing proofs of the Coleman-Mandula theorem shows that they make essential use of a continuity assumption,* and so apply only to the case of a continuous internal symmetry group. To see whether it is reasonable to postulate their extension to the discrete symmetry case, we have tried to construct a local Lagrangian counterexample. Consider, for example a Lagrangian density containing the terms

$$\Psi^\dagger_{ML}\Phi^{2M+1}_L + \Psi^\dagger_{NL}\Phi^{2N+1}_L + ... \quad , \tag{C2a}$$

with Lorentz indices contracted with each other or with partial derivatives to form a Lorentz scalar. When the auxiliary field Φ_L undergoes a discrete chiral transformation

$$\Phi_L \rightarrow \Phi_L \exp(i\alpha) \quad , \tag{C2b}$$

the invariance of the Lagrangian of Eq. (C2a) requires that Ψ_{ML} transform as

$$\Psi_{ML} \rightarrow \Psi_{ML} \exp[i2\pi(2M+1)\alpha] \quad , \tag{C2c}$$

and similarly for Ψ_{NL}. Thus the existence of a Lagrangian of the form of Eq. (C2a) would give a local Lagrangian example of the behavior leading to the off-diagonal mass term of Eq. (C1a). However, since Φ_L has only two components and is a local fermion field, any power of this field higher than the second vanishes, and so Eq. (C2a) contains the two indicated terms only when both M and N are zero, which does not lead to any of

* I am indebted to L. P. Horwitz for email correspondence on this point.

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the off-diagonal couplings discussed above. To avoid having high powers of the auxiliary field Φ_L, one could try to introduce multiple auxiliary fields, and to construct a Lagrangian multilinear in these auxiliary fields so that the auxiliary fields are all forced to have the same chiral transformation of Eq. (C2b) and Ψ_{ML} is forced to have the chiral transformation of Eq. (C2c). The problem here is that by introducing multiple auxiliary fields one introduces the possibility of additional symmetries of the Lagrangian, that typically act to spoil the counterexample. For example, consider the Lagrangian (with all fields now understood to be left handed, so the subscript L is suppressed)

$$A\Phi_a^\dagger\Phi_c + B\Phi_b^\dagger\Phi_c + C\Phi_a^\dagger\Phi_b + D\Psi_1^\dagger\Phi_a\Phi_b\Phi_c \; + \; \text{adjoint} \quad , \qquad (C3a)$$

that would seem to have all properties needed to force Ψ_1 to obey Eq. (C2c) with $M = 1$. But redefining $\Phi_a \to A^{*-1}\Phi_a$ and $\Phi_b \to B^{*-1}\Phi_b$, Eq. (C3a) takes the form

$$\Phi_a^\dagger\Phi_c + \Phi_b^\dagger\Phi_c + C'\Phi_a^\dagger\Phi_b + D'\Psi_1^\dagger\Phi_a\Phi_b\Phi_c \; + \; \text{adjoint} \quad . \qquad (C3b)$$

The first three terms of Eq. (C3b) can be rewritten (including adjoints) as

$$(\Phi_a + \Phi_b)^\dagger\Phi_c \; + \; \text{adjoint} + \frac{C'}{2}[(\Phi_a + \Phi_b)^\dagger(\Phi_a + \Phi_b) - (\Phi_a - \Phi_b)^\dagger(\Phi_a - \Phi_b)] \quad , \qquad (C3c)$$

showing that the chiral phase of $\Phi_a - \Phi_b$ is in fact not restricted by these terms. Thus the Lagrangian of Eq. (C3a) has a two parameter, rather than a one parameter, chiral symmetry group and so does not provide a suitable counterexample. The above arguments are not systematic enough to constitute a proof, but the fact that it does not seem easy to make a local Lagrangian counterexample suggests that the Coleman-Mandula theorem may be extendable to discrete chiral symmetries. This is an interesting question for further study.

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Figure Caption

Fig. 1 Matching of the two branches of the running coupling across the nonperturbative regime around Λ_H^2. The match is parameterized by g_*^2, the value of the coupling at the intersection of the extrapolated $g_{SU(3)}^2$ of branch 2 with $g_{SU(4)}^2$ of branch 1. The magnitude of g_*^2 is determined by the physics of bound state formation and gauge charge screening in the nonperturbative regime.